1.1 Modulus Functions

Question Paper

Course	CIE A Level Maths
Section	1. Algebra & Functions
Topic	1.1 Modulus Functions
Difficulty	Hard

Time allowed: 60

Score: /45

Percentage: /100

Question 1a

(a) On the same axes, sketch the graphs of $y = |f(x)|$ and $y = |g(x)|$ where

$$f(x) = 3x - 1 \qquad x \in \mathbb{R}$$
$$g(x) = 2x + 2 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

[3 marks]

Question 1b

(b) Solve the equation $|f(x)| = |g(x)|$.

[3 marks]

Question 1c

(c) Which of the solutions to $|f(x)| = |g(x)|$ is also a solution to $f(x) = g(x)$?

[1 mark]

Question 2a

The function $f(x)$ is defined as

$$f: x \mapsto |3x - 2| \qquad x \in \mathbb{R}$$

(a) Explain why the inverse of $f(x)$ does not exist.

[1 mark]

Question 2b

(b) Suggest an adaption to the domain of $f(x)$ so its inverse does exist, but also produces the maximum possible range for $f(x)$.

[1 mark]

Question 2c

(c) Using your adaption from part (b), find an expression for $f^{-1}(x)$ and state its domain and range.

[3 marks]

Question 3

Solve the equation $|x^2 - 4| = 3$, giving your answers in exact form.

[3 marks]

Question 4a

The functions $\mathrm{f}(x)$, $\mathrm{g}(x)$ are defined as follows

$$\mathrm{f}(x) = |x - 2| - 5 \qquad x \in \mathbb{R}$$
$$\mathrm{g}(x) = |x| \qquad x \in \mathbb{R}$$

(a) Sketch the graph of $y = \mathrm{gf}(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

[4 marks]

Question 4b

(b) (i) How many solutions are there to the equation $\mathrm{gf}(x) = 1$?
(ii) How many solutions are there to the equation $\mathrm{gf}(x) = 10$?

[2 marks]

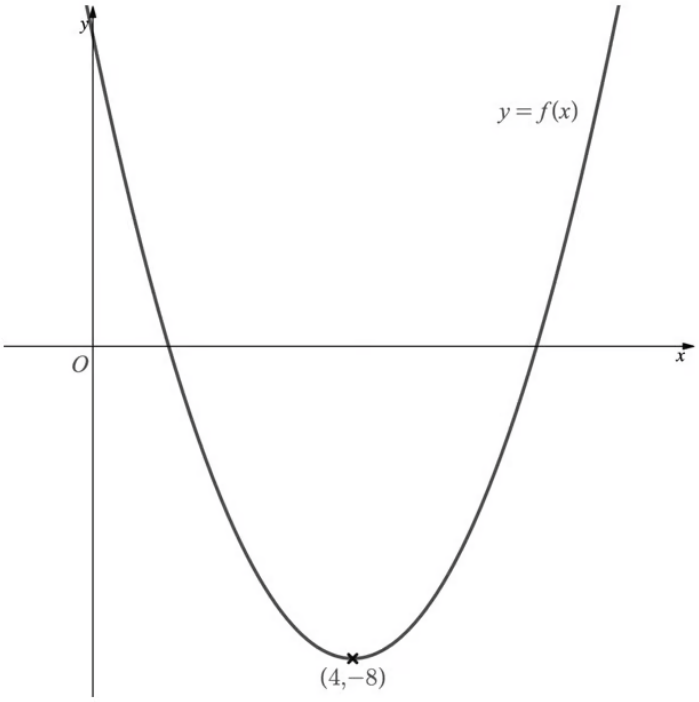
Question 4c

(c) Solve the equation $\mathrm{gf}(x) = 2$.

[3 marks]

Question 5a

The minimum point on the graph of $y = \mathrm{f}(x)$ has coordinates $(4\ , -8)$ as shown on the diagram below.



(a) Sketch the graph of $y = |\mathrm{f}(2x)| - 3$ and state the coordinates of the maximum point.

[3 marks]

Question 5b

(b) Find the exact distance between the minimum point on the graph of $y = \mathrm{f}(x)$ and the maximum point on the graph of $y = |\mathrm{f}(2x)| - 3$.

[2 marks]

Question 6a

(a) On the same axes sketch the graphs of $y = \mathrm{p}(x)$ and $y = p^{-1}(x)$, where $\mathrm{p}(x) = |2x|, \ x \leq 0$.

[3 marks]

Question 6b

(b) Find an expression for $\mathrm{p}^{-1}(x)$ and state its domain.

[3 marks]

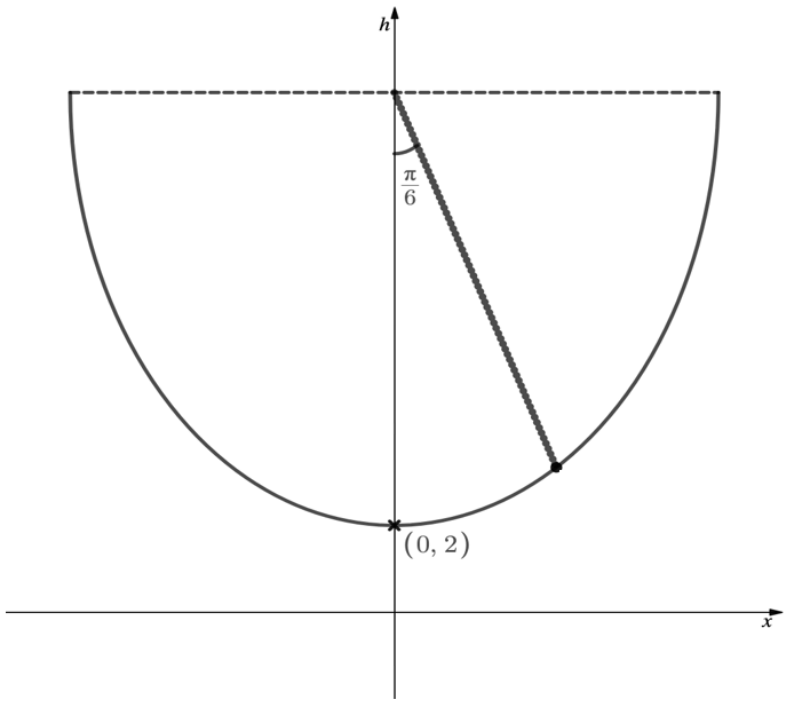
Question 6c

(c) Show that $\mathrm{p}^{-1}(x) = -\frac{1}{2}\mathrm{p}(-\frac{1}{2}x)$.

[3 marks]

Question 7a

The path of a swing boat fairground ride that swings forwards and backwards is modelled as a semi-circle, radius 10 m, as shown in the diagram below.



At time t seconds, the x-coordinate of the boat is modelled by the function

$$x(t) = 10 \sin\left(\frac{\pi}{5}t\right), \ t \geq 0,$$

and the height, h m, of the boat above the ground, at time t seconds, is modelled by

$$h(t) = 12 - 10\left|\cos\left(\frac{\pi}{5}t\right)\right|, \ t \geq 0.$$

(a) Verify that the initial position of the boat is $(0\ , 2)$.

[2 marks]

Question 7b

(b) (i) Write down the coordinates of the boat when it is at its maximum height.
(ii) Find the time it takes the boat to swing between these two points.

[3 marks]

Question 7c

(c) Find the position of the boat when it has swung through an angle of $\frac{\pi}{6}$ anticlockwise from the y-axis, as shown in the diagram above.
Find the time at which the boat first reaches this position.

[2 marks]